Financial Analysis of Power Plant Projects under Market Risk

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Abstract

High market risks are embedded in mega-scale projects. These risks include political instability, economic instability, social risks, technical risks, and other non-financial factors. All these risk factors will have directly impacted on financial feasibility of projects. Hence, it is necessary to perform an elaborate financial analysis of projects at planning stage. A jump diffusion option-pricing model is derived for considering the managerial flexibility of scale expansion in the financial analysis of projects to increase the project value.

Keyword: jump diffusion option pricing model, the real option, call option.

INTRODUCTION

Options can be either call or put. A call is a financial instrument that gives its owner the right, but not the obligation, to purchase the underlying asset (stocks, stock indices, etc.) at a specified price (strike or exercise price) for a specified time. A put option gives its owner the right to sell the underlying at the strike price for a specified time. There are two kinds of options: the American option can be exercised at any time before or at the expiration; the European option can be exercised only at the expiration. We shall only deal with European options. The buyer of an option pays cash the option price to the seller (or writer) who assumes all the obligations of the contract (all the rights are of the buyer).

The term "real options" was coined by Stewart Myers in 1977. It referred to the application of option pricing theory to the valuation of non-financial or "real" investments with learning and flexibility, such as multi-stage R&D, modular manufacturing plant expansion and the like. (Myers, 1977) The topic attracted moderate, primarily academic, interest in the 1980's and 1990's, and a number of articles were published on theory and applications.

Beginning in the mid-1990's, interest in the concepts of value and the techniques of valuation increased substantially. Real options began to attract considerable attention from industry as a potentially important tool for valuation and strategy. Beginning in the oil and gas industry and extending to a range of other industries, management consultants and internal analysts began to apply real options intermittently, and in some cases regularly, to major corporate investment issues. An annual real options symposium for both academics and practitioners was first organized in 1996, and continues to this day. Several practitioner books on the topic, many simply titled *Real Options*, have appeared, and more are in the works. Most mainstream

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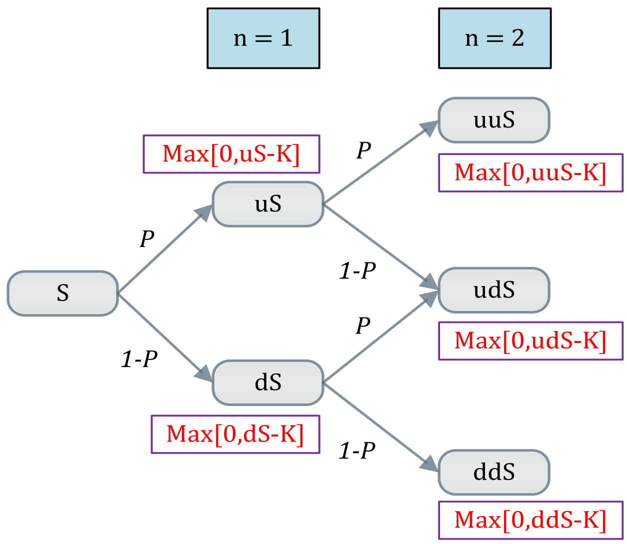
academic finance texts now mention real options prominently. Conferences on the topic, with both academic and industry participants, are held regularly. The increasing number of academic articles on real options is now matched by an increasing number of stories in such mainstream publications as *Business Week* and *USA Today*. All in all, real options has made a transition from a topic of modest academic interest to considerable, active academic and industry attention.

MODELING

Let a stand for the minimum number of upward moves that the stock must make over the next n periods for the call to finish in-the-money. Thus, a will be the smallest non-negative integer such that $u^a d^{n-a} S > K$. By taking the natural logarithm of both sides of this inequality, we could write a as the smallest non-negative integer greater than $\log(K/Sd^n)/\log(u/d)$. For all $j < a$,



$\max[0, u^j d^{n-j} S - K] = 0$, and for all $j \geq a$,
$\max[0, u^j d^{n-j} S - K] = u^j d^{n-j} S - K$
Therefore,

$$C = \left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} [u^j d^{n-j} S - K] \right] / r^n$$

By breaking up C into two terms, we can write

$$C = S\left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \left(\frac{u^j d^{n-j}}{r^n} \right) \right] - Kr^{-n} \left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^{n-j} \right]$$

Now, the latter bracketed expression is the complementary binomial distribution function $\phi[a; n, p]$. The first bracketed expression can also be interpreted as a complementary binomial distribution function $\phi[a; n, p']$, where

$$p' \equiv (u/r)p \text{ and } 1 - p' \equiv (d/r)(1-p)$$

p' is a probability, since $0 < p' < 1$. To see this, note that $p < (r/u)$ and

$$p^j(1-p)^{n-j}\left(\frac{u^j d^{n-j}}{r^n}\right)=\left[\frac{u}{r}p\right]^j=\left[\frac{d}{r}(1-p)\right]^{n-j}=p'^j(1-p')^{n-j}$$

The bi-nominal model is in the following form.

$$\mathrm{C}=\mathrm{S}\Phi[a;n,p']-Kr^{-n}\Phi[a;n,p]$$

where

$$\mathrm{p}\equiv\frac{\mathrm{r}-\mathrm{d}}{\mathrm{u}-\mathrm{d}}\text{ and }\mathrm{p}'\equiv\left(\frac{\mathrm{u}}{\mathrm{r}}\right)\mathrm{p}$$

$$\mathrm{a}\equiv\text{the smallest non}-\text{negative integer}$$

$$\mathrm{a}\geq\log\left(\frac{\mathrm{k}}{\mathrm{Sd}'}\right)/\log(\frac{\mathrm{u}}{\mathrm{d}})$$

If a>n, then c=0. With

$$\Phi[a;n,p']=\sum_{j=a}^{n}\left[\frac{n!}{j!\,(n-j)!}p^j(1-p)^{n-j}\left(\frac{u^j d^{n-j}}{r^n}\right)\right]$$

$$\Phi[a;n,p]=\sum_{j=a}^{n}\left[\frac{n!}{j!\,(n-j)!}p^j(1-p)^{n-j}\right]$$

The derivation of parameter u, d, P in diffusion Model (Black-Sholes Model) as follows:

$$\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)=j\mathrm{logu}+(\mathrm{n}-j)\mathrm{logd}=j\log\left(\frac{\mathrm{u}}{\mathrm{d}}\right)+\mathrm{n\ logd}$$

$$\mathrm{E}[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)]=\log(\frac{\mathrm{u}}{\mathrm{d}})\times\mathrm{E}(j)\ +\mathrm{n\ logd}$$

$$\mathrm{Var}\left[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)\right]=\left[\log\left(\frac{\mathrm{u}}{\mathrm{d}}\right)\right]^2\times\mathrm{Var}(j)$$

$$\mathrm{E}(j)=\mathrm{nq}$$

$$\mathrm{Var}(j)=\mathrm{nq}(1-\mathrm{q})$$

$$\mathrm{E}[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)]=[\mathrm{qlog}(\frac{\mathrm{u}}{\mathrm{d}})\ +\ \mathrm{logd}]*\mathrm{n}\ \equiv\ \hat{\mu}\mathrm{t}$$

$$\mathrm{Var}\left[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)\right]=\mathrm{q}\ (1-\mathrm{q})\left[\log\left(\frac{\mathrm{u}}{\mathrm{d}}\right)\right]^2*\mathrm{n}\ \equiv\ \hat{\sigma}^2\mathrm{t}\ \text{ as n}\rightarrow\infty$$

$$\mathrm{E}\left[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)\right]=\left[\mathrm{qlog}\left(\frac{\mathrm{u}}{\mathrm{d}}\right)+\ \mathrm{logd}\right]*\mathrm{n}\ \rightarrow\ \mu\mathrm{t}\ \text{ as n}\rightarrow\infty$$

$$\mathrm{Var}\left[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)\right]=\mathrm{q}\ (1-\mathrm{q})\left[\log\left(\frac{\mathrm{u}}{\mathrm{d}}\right)\right]^2*\mathrm{n}\ \rightarrow\ \sigma^2\mathrm{t}$$

$$\mathrm{E}[\log\left(\frac{\mathrm{S}^*}{\mathrm{S}}\right)]=[\mathrm{qlog}(\frac{\mathrm{u}}{\mathrm{d}})\ +\ \mathrm{logd}]*\mathrm{n}\ =\ \mu\mathrm{t} \qquad (1)$$

$$\mathrm{Var}\left[\log\left(\frac{S^*}{S}\right)\right] = q\,(1-q)\left[\log\left(\frac{u}{d}\right)\right]^2 * n \;=\; \sigma^2 t \qquad (2)$$

Rearrange the equation 1. We obtain

$$q\log\left(\frac{u}{d}\right) + \mathrm{logd} = \frac{\mu t}{n}$$

$$q\log\left(\frac{u}{d}\right) = \frac{\mu t}{n} - \mathrm{logd}$$

$$q = (\mu\frac{t}{n} - \mathrm{logd})/\log\left(\frac{u}{d}\right)$$

$$q = \frac{\mu\left(\frac{t}{n}\right) - \mathrm{logd}}{\mathrm{logu} - \mathrm{logd}} \qquad (3)$$

$$1 - q = \frac{\mathrm{logu} - \mathrm{logd}}{\mathrm{logu} - \mathrm{logd}} - \frac{\mu\left(\frac{t}{n}\right) - \mathrm{logd}}{\mathrm{logu} - \mathrm{logd}} = \frac{\mathrm{logu} - \mu\left(\frac{t}{n}\right)}{\mathrm{logu} - \mathrm{logd}} \qquad (4)$$

Insert equation (3) and (4) into equation (2), we have

$$\frac{\mu\left(\frac{t}{n}\right) - \mathrm{logd}}{\mathrm{logu} - \mathrm{logd}} \times \frac{\mathrm{logu} - \mu\left(\frac{t}{n}\right)}{\mathrm{logu} - \mathrm{logd}} \times \left[\log\left(\frac{u}{d}\right)\right]^2 = \sigma^2\frac{t}{n}$$

$$\frac{(\mu\left(\frac{t}{n}\right) - \mathrm{logd}) \times (\mathrm{logu} - \mu\left(\frac{t}{n}\right))}{[\mathrm{logu} - \mathrm{logd}]^2} \times [\log u - \mathrm{logd}]^2 = \sigma^2\frac{t}{n}$$

$$(\mu\left(\frac{t}{n}\right) - \mathrm{logd}) \times (\mathrm{logu} - \mu\left(\frac{t}{n}\right)) = \sigma^2\frac{t}{n}$$

Let $u = \frac{1}{d}$, we have $\mathrm{logd} = \log\frac{1}{u} = -\mathrm{logu}$

$$(\mathrm{logu} + \mu\left(\frac{t}{n}\right)) \times (\mathrm{logu} - \mu\left(\frac{t}{n}\right)) = \sigma^2\frac{t}{n}$$

$$(\mathrm{logu})^2 - [\mu\left(\frac{t}{n}\right)]^2 = \sigma^2\frac{t}{n}$$

$$(\mathrm{logu})^2 = \sigma^2\frac{t}{n} + \mu^2\left(\frac{t}{n}\right)^2$$

$$\mathrm{logu} = [\sigma^2\frac{t}{n} + \mu^2\left(\frac{t}{n}\right)^2]^{1/2}$$

$$\mathrm{logu} = \sigma\sqrt{\frac{t}{n}} \times [1 + \left(\frac{\mu}{\sigma}\right)^2 (\frac{t}{n})]^{\frac{1}{2}}$$

$$\because (1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{2\times 4}x^2 + \cdots\cdots$$

$$\therefore [1 + \left(\frac{\mu}{\sigma}\right)^2 (\frac{t}{n})]^{\frac{1}{2}} = 1 + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\left(\frac{t}{n}\right) - \cdots\cdots \approx 1$$

$$\log u = \sigma\sqrt{\frac{t}{n}}$$

$$u = e^{\sigma\sqrt{\frac{t}{n}}}$$

$$\because \log d = \log\frac{1}{u} = -\log u$$

$$\therefore q = \frac{\mu\left(\frac{t}{n}\right) - \log d}{\log u - \log d} = \frac{\mu\left(\frac{t}{n}\right) + \log u}{\log u + \log u} = \frac{\mu\left(\frac{t}{n}\right) + \sigma\sqrt{\frac{t}{n}}}{\sigma\sqrt{\frac{t}{n}} + \sigma\sqrt{\frac{t}{n}}} = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\frac{t}{n}}$$

The parameters for continuous diffusion model are then obtained as follows:

$$u = \frac{1}{d} = e^{\sigma\sqrt{\frac{t}{n}}} \tag{5}$$

$$d = e^{-\sigma\sqrt{\frac{t}{n}}} \tag{6}$$

$$q = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu}{\sigma}\right)\sqrt{\frac{t}{n}} \tag{7}$$

A Jump-Diffusion Model can be then derived in same fashion.

$$d = e^{\zeta(\frac{t}{n})},\ \ q = \lambda(\frac{t}{n})$$

ζ is the jump size, which follows normal distribution. λ is average number of jumps per unit time, which follows Poisson process.

$$u = u, \qquad d = e^{\zeta(\frac{t}{n})},\ \ q = \lambda(\frac{t}{n})$$

$$\Psi[x; y] = \sum_{i=x}^{\infty}\frac{e^{-y}y^i}{i!}$$

$$C = S\,\Psi\,[x; y] - Kr^{-t}\,\Psi\left[x; \frac{y}{u}\right]$$

Where

$$y = \left(\log r - \zeta\right)ut/(u-1)$$

and $x = \text{the mallest non} - \text{negative integer greater than } \frac{\left(\log\left(\frac{K}{S}\right) - \zeta\, t\right)}{\log u}$。

CONCLUSIONS

Investment timing is always a critical issue to consider for investing BOT or Private-Public-Participation (PPP) projects. A jump-diffusion option pricing model for different investment time of projects is established in this study. We would apply this model to analyze a power plant project in the future. .

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